Swing-up control based on virtual composite links for $n$-link underactuated robot with passive first joint

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Abstract

This paper concerns the swing-up control of an $n$-link revolute robot moving in the vertical plane with the first joint being passive and the others being active. The goal of this study is to design and analyze a swing-up controller that can bring the robot into any arbitrarily small neighborhood of the upright equilibrium point with all links in the upright position. To achieve this challenging objective while preventing the robot from becoming stuck at an undesired closed-loop equilibrium point, we first address the problem of how to iteratively devise a series of virtual composite links to be used for designing a coordinate transformation on the angles of all the active joints. Second, we devise an energy-based swing-up controller that uses a new Lyapunov function based on that transformation. Third, we analyze the global motion of the robot under the controller and establish conditions on the control parameters that ensure attainment of the swing-up control objective; specifically, we determine the relationship between the closed-loop equilibrium points and a control parameter. Finally, we verify the theoretical results by means of simulations on a 4-link model of a gymnast on the high bar. This study not only unifies some previous results for acrobots and three-link robots with a passive first joint, but also provides insight into the energy- and passivity-based control of underactuated multiple-degree-of-freedom systems.

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1. Introduction

The last two decades have witnessed considerable progress in the study of underactuated robots, which have fewer actuators than degrees of freedom. The mechanisms of the robots can be categorized into different types, e.g., 1) intentionally designed pendulum-type robots (Hauser & Murray, 1990; Spong, 1995; Spong & Block, 1995); 2) rigid robots with elastic joints or flexible links (Spong, 1998); and 3) robots with actuator failure (Arai, Tanie, & Shiroma, 1998). Since these robots usually have nonholonomic second-order constraints, the control problems are challenging (see e.g., Acosta, Ortega, Astolfi, & Mahindrakar, 2005; Ge, Wang, & Lee, 2003; Grizzle, Moog, & Chevallereau, 2005; Jiang, 2002; Lin, Pongvuthithum, & Qian, 2002; Ortega, Spong, Gomez-Estern, & Blankenstein, 2002; Reyhanoglu, van der Schaft, McClamroch, & Kolmanovsky, 1999).


Spong (1996) studied swing-up control for a 3-link planar robot in a vertical plane with a passive first joint; however, there has been no report on a complete analysis of the behavior of the resulting closed-loop system. For this robot, Xin and Kaneda (2007b) showed that, unlike 2-DOF underactuated robots (Fantoni et al., 2000; Kolesnichenko & Shiriaev, 2002; Xin & Kaneda, 2007a), it is difficult to analyze the motion when it is governed by a swing-up controller employing a conventional Lyapunov function, which contains the energy of the robot, and the angles and angular velocities of the two active joints. To overcome this difficulty, Xin and Kaneda (2007b) treated the 2nd and 3rd links as a virtual...
composite link and devised a coordinate transformation on the angles of the two active joints. Moreover, the transformation was used to design an energy-based swing-up controller and the motion of the 3-link robot under that controller was analyzed.

There is increasing interest in constructing multi-link robots that can jump, walk, do gymnastics, etc. (see, e.g., Arikawa & Mita, 2002; Hyon, Yokoyama, & Emura, 2006; Shkolnik & Tedrake, 2008). These studies will not only improve the mobility of humanoid robots, but also give rise to new types of robots.

Since there are few reports on the design and analysis of controllers for general n-DOF underactuated robots (De Luca & Oriolo, 2002), in this study we investigated how to extend the design and analysis of swing-up control for the 3-link robot in Xin and Kaneda (2007b) to a general n-link planar robot with a passive first joint. To guarantee that the robot enters the basin of attraction of any locally stabilizing controller for the upright equilibrium point, where all links are in the upright position, we aimed to design and analyze a swing-up controller that drives the robot from any initial state into any arbitrarily small neighborhood of the equilibrium point.

To achieve this challenging objective, we need to prevent an n-link robot from becoming stuck at an undesired closed-loop equilibrium point. In contrast to the 3-link case, when n > 3, there are many ways of combining n − 1 active links into virtual composite links. Our idea for clarifying the structures of the closed-loop equilibrium configurations is to iteratively study n − 1 robots of the acrobot type (that is, two links with a passive first joint) rather than directly studying an n-link robot. Specifically, from an analysis of the equilibrium points of an n-link robot, we devise a virtual-composite-link formulation that describes the n-link robot in a way similar to the way an acrobot is described.

In this paper, we first address the problem of how to devise a series of virtual composite links in an iterative manner as a basis for designing a coordinate transformation on the angles of all the active joints. This is one key contribution of this paper. Second, we construct a Lyapunov function based on the transformation and use it to devise a swing-up controller. Third, we analyze the global motion of the robot under the controller; and we establish conditions on two control parameters that eliminate controller singularities and ensure attainment of the swing-up equilibriums. This is another key contribution of this paper. Section 4 describes a swing-up controller for a robot. Section 5 analyzes the global motion of the robot under that controller. Section 6 shows simulation results for a 4-link model. Section 7 makes some concluding remarks.

2. Background and problem formulation

2.1. Model of n-link underactuated robot

Consider an n-link revolute robot moving in the vertical plane with a passive first joint (Fig. 1). For the ith (i = 1, . . . , n) link, mi is its mass, li is its length, li is the distance from joint i to the center of mass (COM) of the ith link, and Ji is the moment of inertia around its COM.

Let q = [q1, q2, . . . , qn]T be the vector of the angles of all the joints in generalized coordinates. In this paper, the angle of the passive joint, q1, is dealt with in S1, which denotes a unit circle, while the vector of the angles of all the active joints, q0 = [q2, . . . , qn]T, is dealt with in ℝn−1. The motion equation of the robot is

\[ M(q)\ddot{q} + H(q, \dot{q}) + G(q) = \tau, \]

where \( M(q) \) ∈ ℝn×n is a symmetric positive definite inertia matrix; \( H(q, \dot{q}) \) ∈ ℝn contains the Coriolis and centrifugal terms; \( G(q) \) ∈ ℝn contains the gravitational terms; and \( \tau \), the input torque vector produced by the n − 1 actuators at active joints 2, . . . , n.

Define the potential energy of the robot to be

\[ P(q) = \sum_{i=1}^{n} m_i g Y_{G_i}, \]

where g is the acceleration of gravity and \( Y_{G_i} \) is the Y-coordinate of the COM of link i, which is given by \( Y_{G_i} = l_{i-1} \cos q_i \), and for 2 ≤ i ≤ n

\[ Y_{G_i} = l_1 \cos q_1 + \cdots + l_{i-1} \cos \sum_{j=1}^{i-1} q_j + l_i \cos \sum_{j=1}^{i} q_j. \]

We write P(q) in terms of \( \cos \sum_{j=1}^{i} q_j \) as

\[ P(q) = \sum_{i=1}^{n} \beta_i \cos \sum_{j=1}^{i} q_j, \]

where \( \beta_i \) is defined to be

\[ \beta_i := m_i l_i g + \sum_{j=i+1}^{n} m_j l_j g, \quad i = 1, \ldots, n-1, \]

\[ \beta_n := m_n l_n g. \]

\( G_i(q), \) the ith element of G(q), is given by

\[ G_i(q) = \frac{\partial P}{\partial q_i} = -\sum_{k=i}^{n} \beta_k \sin \sum_{j=1}^{k} q_j. \]

Furthermore, the total mechanical energy of the robot, E(q, \dot{q}), is

\[ E(q, \dot{q}) = \frac{1}{2} \dot{q}^T M(q) \dot{q} + P(q). \]
From Eq. (30) in Ortega and Spong (1989), we obtain the following equation, which we will use later:
\[ \dot{E} = q^T B \tau = \dot{q}_a^T \tau. \] (9)

2.2. Problem formulation

Consider the upright equilibrium point
\[ q = 0, \quad \dot{q} = 0. \] (10)

The potential energy, \( E_r \), of the robot at that point is \( E_r = \sum_{i=1}^{n} b_i \).

To solve the problem we are studying, we first need to determine whether or not control input \( \tau \) can be designed such that
\[ \lim_{t \to \infty} E_r(q, \dot{q}) \leq 0, \quad \lim_{t \to \infty} \dot{q}_a = 0, \quad \lim_{t \to \infty} q_a = 0. \] (11)

Section 5.1 shows that, if (11) holds, then the goal of swinging the robot up into any arbitrarily small neighborhood of the upright equilibrium point is achieved.

Now, we explain why it is difficult to analyze the motion of the robot under a controller designed using the following conventional Lyapunov function candidate:
\[ V_C = \frac{1}{2} (E - E_r)^2 + \frac{1}{2} k_0 \dot{q}_a^2 + \frac{1}{2} k_0 q_a^2, \] (12)

where \( k_0 \in \mathbb{R} \) and \( k_0 \in \mathbb{R} \) are positive constants. From (9), the time derivative of \( V_C \) along the trajectory of (1) is
\[ \dot{V}_C = \dot{q}_a^T (E - E_r) \tau + k_0 \dot{q}_a q_a. \]

If we can choose \( \tau \) such that
\[ (E - E_r) \tau + k_0 \dot{q}_a q_a = -k_0 \dot{q}_a \] (13)

for a scalar constant \( k_0 > 0 \), then
\[ \dot{V}_C = -k_0 \dot{q}_a^2 q_a \leq 0. \] (14)

To complete the motion analysis of the robot under a controller satisfying (13), it is crucial to know the robot from becoming stuck at an undesired closed-loop equilibrium point. Let \( q^e = [q_1^e, q_2^e, \ldots, q_n^e]^T \) be a closed-loop equilibrium configuration. From (1), (13), and \( q(t) \equiv q^e \), we obtain
\[ G_t(q^e)^T = 0, \] (15)
\[ k_0 q_i^e + (P(q^e) - E_r) \tau_i^e = 0, \quad 2 \leq i \leq n, \] (16)

where \( \tau_i^e = G_t(q^e) \) is the equilibrium torque. It is easy to see that
\[ q^e = [0, 0, \ldots, 0] \] (the upright equilibrium configuration) and
\[ q^e = [\pi, 0, \ldots, 0]^T \] (the downward equilibrium configuration), for which \( \tau_i^e = 0 \), are solutions to (15) and (16) for any given \( k_p \). However, it is difficult to predict conditions on \( k_p \) that ensure that (15) and (16) do not have other solutions. (See a further explanation in Section 5.3.)

To determine the relationship between the control parameter and the closed-loop equilibrium configurations, rather than studying an \( n \)-link robot, we are interested in iteratively studying \( n - 1 \) acrobot-type robots. (Note: An acrobot is a two-link robot with a passive first joint that has already been studied a great deal.) To this end, we present the notion of virtual composite link, which enables us to devise a new Lyapunov function for designing \( \tau \) and to analyze the global motion of the robot.

3. Virtual composite links and coordinate transformation

For links 2 to \( n \) of the robot in Fig. 1, we define a series of virtual composite links (VCLs) as follows: for \( i = 2, \ldots, n \), VCL \( i \) is a single link which starts at joint \( i \) and has the same mass and COM as the group of \( n - i - 1 \) links from links \( i \) to \( n \).

Although defining VCL \( n \) to be link \( n \) somewhat abides the idea of "virtual", such a definition allows us to iteratively construct a series of VCLs: for \( i = 2, \ldots, n - 1 \), VCL \( i \) is a composite link consisting of link \( i \) and VCL \( i + 1 \) (Fig. 2). This facilitates expression of the results in this paper.

For VCL \( i \) in Fig. 2, we define \( \bar{q}_i \) to be the angle between link \( i - 1 \) and a line from joint \( i \) to the COM of VCL \( i \), and \( \theta_{i+1} \) to be the angle between link \( i \) and a line from joint \( i \) to the COM of VCL \( i \). Moreover, when link \( i \) and VCL \( i + 1 \) are stretched out in a straight line \( (\bar{q}_{i+1} = 0) \), it is reasonable to define
\[ \theta_{i+1} = 0, \quad \text{when} \quad \bar{q}_{i+1} = 0, \quad \text{for} \quad 2 \leq i \leq n - 1. \] (17)

Define
\[ \bar{q}_a := [\bar{q}_2, \ldots, \bar{q}_n]^T \in \mathbb{R}^{n-1}. \] (18)

We now give the transformation \( T : \mathbb{R}^{n-1} \to \mathbb{R}^{n-1} \) of \( q_a \) to \( \bar{q}_a \):
\[ \bar{q}_a = T(q_a). \] (19)

Below, we show that \( T(q_a) \) in (19) is given by
\[ \bar{q}_a = 0 \iff q_a = 0, \] (20)
\[ \bar{q}_a = \Psi(q_a) q_a, \] (21)

where \( \Psi(q_a) \in \mathbb{R}^{(n-1) \times (n-1)} \) is the upper triangular matrix
\[
\Psi(q_a) = \begin{cases}
1 & \psi_{23} & \cdots & \psi_{2n} \\
0 & 1 & \cdots & \psi_{3n} \\
0 & 0 & \cdots & 1 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1
\end{cases},
\] (22)

where \( \psi_{ij} (i = 2, \ldots, n, j = 3, \ldots, n) \) are described below.

First, for \( 2 \leq i \leq n \), we use Fig. 2, the fact that \( \bar{q}_i \) is the angle of link \( i \) with respect to link \( i - 1 \), and the definitions of \( \bar{q}_i \) and \( \theta_{i+1} \) to obtain
\[
\begin{align*}
\bar{q}_i &= \bar{q}_{i+1} + \theta_{i+1}, \quad \text{for} \quad 2 \leq i \leq n - 1, \\
\bar{q}_n &= q_n.
\end{align*}
\] (23)

Thus, (20) is a direct consequence of (17).

Next, to prove (21), we use a Cartesian coordinate system \((X_i, Y_i)\) with the origin at joint \( i \) and the \( x \)-axis lying along link
i (Fig. 3) to determine $\theta_{t+1}$. Let $l_{ij}$ be the distance between joint $i$ and the COM of VCL $i$. Then, the COMs of link $i$ and VCL $i + 1$ have the coordinates $(l_{ij}, 0)$ and $(l_{ij} + l_{ij+1}, l_{ij+1} \sin \beta_i)$, respectively. Let $(x_{ci}, y_{ci})$ be the coordinates of the COM of VCL $i$. Then, combining the COM of link $i$ and the COM of VCL $i + 1$ to obtain the COM of VCL $i$ yields

$$ (x_{ci}, y_{ci}) = \left( \frac{\beta_i + \beta_i \cos \beta_i, \beta_i \sin \beta_i}{m_i g} \right), $$

where $m_i := \sum_{j=1}^{n} m_j$ is the mass of VCL $i$, and

$$ \tilde{p}_i := \bar{m}_{ic} g. $$

Next, from Fig. 3 and (24), we obtain

$$ \begin{aligned}
\sin \theta_{t+1} &= \frac{y_{ci}}{l_{ci}} = \frac{\tilde{p}_i \sin \tilde{q}_i}{\tilde{p}_i}, \\
\cos \theta_{t+1} &= \frac{x_{ci}}{l_{ci}} = \frac{\beta_i + \alpha_i \cos \beta_i}{\tilde{p}_i}.
\end{aligned} $$

Since $\theta_{t+1}$ is dealt with in $\mathbb{R}$ rather than in $S^1$, we cannot determine it just from the values of its sine and cosine in (26). In fact, $\theta_{t+1}$ must also satisfy (17), which is why we need to obtain $\theta_{t+1}$. Using the relation

$$ \dot{\theta}_{t+1} = \frac{d(\sin \theta_{t+1})}{dt} \cos \theta_{t+1} - \frac{d(\cos \theta_{t+1})}{dt} \sin \theta_{t+1} $$

and (26), we have

$$ \dot{\theta}_{t+1} = w_{i+1} \dot{q}_{i+1} + v_{i+1} \dot{\tilde{q}}_{i+1}, \quad \text{for} \ 2 \leq i \leq n - 1, $$

where

$$ w_{i+1} := \frac{\bar{p}_i (\bar{p}_i + \beta_i \cos \bar{q}_i)}{\beta_i^2}, \quad v_{i+1} := \frac{\beta_i \sin \bar{q}_i}{\beta_i}. $$

This, along with (23), gives

$$ \ddot{q}_i = \dot{q}_i + w_{i+1} \ddot{q}_{i+1} + v_{i+1} \dot{\tilde{q}}_{i+1}, \quad \text{for} \ 2 \leq i \leq n - 1. $$

To obtain $\ddot{\beta}_i$ in (29), we use (25) and $l_{ci} = \sqrt{x_{ci}^2 + y_{ci}^2}$ to derive the following iterative relation:

$$ \begin{aligned}
\ddot{\beta}_i &= h(\beta_i, \beta_i, \dot{\tilde{q}}_i), \quad \text{for} \ 2 \leq i \leq n - 1, \\
\ddot{\beta}_n &= \ddot{\beta}_n,
\end{aligned} $$

where

$$ h(a, b, z) := \sqrt{a^2 + b^2 + 2ab \cos z}. $$

Thus, taking the time derivative of $\ddot{\beta}_i$ in (30) yields

$$ \ddot{\beta}_i = \ddot{p}_i \dot{\tilde{q}}_{i+1} + f_{i+1} \ddot{\tilde{q}}_{i+1}, $$

where

$$ \ddot{p}_i := -\frac{\beta_i \ddot{\beta}_i \cos \bar{q}_i}{\ddot{\beta}_i}, \quad f_{i+1} := \frac{\ddot{\beta}_i + \beta_i \cos \bar{q}_i}{\ddot{\beta}_i}. $$

Now, using (29), (32), and the facts $\ddot{\beta}_n = \ddot{\beta}_n = \ddot{\beta}_n$, we have

$$ \ddot{\beta}_{n-1} = \ddot{\beta}_{n-1} + \psi_{(n-1)n} \ddot{q}_n, \quad \ddot{\beta}_n = \phi_{(n-1)n} \ddot{q}_n, $$

where $\psi_{(n-1)n} = \psi_{n}$ and $\phi_{(n-1)n} = \phi_{n}$. Using (29) and (32), we iterate backwards to obtain $\ddot{\beta}_{n-2}, \ddot{\beta}_{n-2}, \ldots, \ddot{\beta}_i, \ddot{\beta}_i,$ and $\ddot{\beta}_i$, and we have

$$ \ddot{\beta}_i = \ddot{\beta}_i + \sum_{j=i+1}^{n} \beta_j \ddot{q}_j, \quad \ddot{\beta}_i = \sum_{j=i+1}^{n} \phi_j \ddot{q}_j, $$

for $2 \leq i \leq n - 1$, where $\beta_j$ and $\phi_j$ are defined to be

$$ \begin{aligned}
\psi_{j+1} &= \psi_{j+1}, \quad \psi_{j+1} = \psi_{j+1} \psi_{(j+1)j} + \psi_{j+1} \psi_{(j+1)j}, \\
\phi_{j+1} &= \phi_{j+1}, \quad \phi_{j+1} = \phi_{j+1} \psi_{(j+1)j} + \phi_{j+1} \psi_{(j+1)j}.
\end{aligned} $$

for $i + 2 \leq j \leq n$. This demonstrates (21).

Finally, if $l_{ci}$ (the distance between joint $i$ and the COM of VCL $i$) is zero, that is, if VCL $i$ shrinks to a point at joint $i$, then neither $\ddot{\beta}_i$ nor $\ddot{\beta}_i$ can be well defined. To prevent this from happening, we make the following assumption.

**Assumption 1.** $l_{ci} > 0$ holds for all $[q_{i+1}, \ldots, q_n]^T \in \mathbb{R}^{n-1}$.

Regarding Assumption 1, from (25) and (30) we know that $l_{ci} = 0$ (or equivalently, $\ddot{\beta}_i \neq 0$) if and only if $\ddot{\beta}_i = \ddot{\beta}_i$ and $\cos \bar{q}_i = -1$. Thus, a sufficient condition that $\ddot{\beta}_i \neq 0$ for all $[q_{i+1}, \ldots, q_n]^T \in \mathbb{R}^{n-1}$ is

$$ \beta_{n-1} \neq \beta_n, \quad \beta_i > \beta_{j+1}, \quad \text{for} \ 2 \leq i \leq n - 2. $$

Indeed, from (30), we obtain $\ddot{\beta}_i \leq |\ddot{\beta}_i - \ddot{\beta}_i|$ and $\ddot{\beta}_i \neq \ddot{\beta}_i$, $\ddot{\beta}_i \leq \ddot{\beta}_i + \ddot{\beta}_i \leq \sum_{j=i+1}^{n} \beta_j$. Thus, $\ddot{\beta}_{n-1} \neq \beta_n$ yields $\ddot{\beta}_n > 0$, while for $2 \leq i \leq n - 2$, the inequality $\ddot{\beta}_i > \sum_{j=i+1}^{n} \beta_j \geq \ddot{\beta}_i + \ddot{\beta}_i$ yields $\ddot{\beta}_i > 0$.

An example of a robot satisfying (37) is a 4-link robot with $m_i = m, l_{ci} = l/2 = l/2$ for $1 \leq i \leq 4$.

### 4. Swing-up controller for n-link robot

The Lyapunov function candidate is

$$ V = \frac{1}{2} (E - E_t)^2 + \frac{1}{2} k_0 \dot{q}_a^2 + \frac{1}{2} k_0 \dot{q}_a^2. $$

We use $q_a$ in $V$ instead of $q_a$ in $V_c$ in (21); and from (20), we know that $\lim_{t \to \infty} V = 0$ is equivalent to (11). Taking the time derivative of $V$ along the trajectories of (1) and using (21), we obtain

$$ \dot{V} = \ddot{q}_a (E - E_t) \tau + k_0 \dot{q}_a + k_0 \dot{q}_a \tau. $$

If we can choose $\tau$ such that

$$ (E - E_t) \tau + k_0 \dot{q}_a + k_0 \dot{q}_a \tau = -k_0 \dot{q}_a $$

for some constant $k_0 > 0$, then we have

$$ \dot{V} = -k_0 \dot{q}_a \dot{q}_a \leq 0. $$
Now, we discuss under what condition (39) is solvable for \( \tau \) for any \((q, \dot{q})\). We obtain \( \tilde{q}_n \) from (1); and substituting it into (39) yields

\[
A(q, \dot{q})\tau = k_B B^T \frac{1}{M(q)}(H(q, \dot{q}) + G(q)) \\
- k_V \tilde{q}_n - k_P \tau^T \tilde{q}_n,
\]

where

\[
A(q, \dot{q}) = (E(q, \dot{q}) - E_r)I_{n-1} + k_B B^T \frac{1}{M(q)}B.
\]

Hence, when

\[
|\Lambda(q, \dot{q})| \neq 0, \quad \forall (q, \dot{q}),
\]

we obtain

\[
\tau = \Lambda^{-1} \left( k_B B^T \frac{1}{M(q)}(H + G) - k_V \tilde{q}_n - k_P \tau^T \tilde{q}_n \right).
\]

Similar to the method in Xin and Kaneda (2007b), we use the fact that \( M(q) \) does not contain \( q_1 \) and is a matrix function of \( q_n \) to derive a necessary and sufficient condition such that (43) holds. Then, we apply LaSalle's theorem (Khalil, 2002) to the closed-loop system consisting of (1) and (44) to determine the largest invariant set that the closed-loop solution approaches. That produces the following theorem, which is given without proof due to page limitations.

**Theorem 1.** Consider the closed-loop system consisting of (1) and (44). Suppose that \( k_B > 0, k_P > 0, \) and \( k_V > 0 \). Then, controller (44) has no singularities for any \((q, \dot{q})\) if and only if

\[
k_D > \max \left\{ (E_r + \mu(q_n)) \lambda_{\text{max}} \left( (B^T B)^{-1} \right) \right\},
\]

where \( \lambda_{\text{max}}(A) \) denotes the largest eigenvalue of \( A \) \( \geq 0 \), and

\[
\mu(q_n) = \left( \sum_{i=1}^{n} \beta_i^2 + 2 \sum_{i=1}^{n-1} \sum_{j>i} \beta_i \beta_j \cos \left( \frac{j}{n} \right) \right)^{1/2}.
\]

In this case, \( \lim_{t \to \infty} V = V^* \), \( \lim_{t \to \infty} E = E^* \), \( \lim_{t \to \infty} q_n = q_n^* \), and \( \lim_{t \to \infty} \tilde{q}_n = \tilde{q}_n^* \), where \( V^*, E^*, q_n^*, \) and \( \tilde{q}_n^* \) are constants. Moreover, as \( t \to \infty \), every closed-loop solution, \( (q(t), \dot{q}(t)) \), approaches the invariant set

\[
W = \left\{ (q, \dot{q}) \mid q_n^* = \frac{2(E^* - P(q_1, q_n^*))}{M_1(q_2^*)} : q_n = q_n^* \right\},
\]

where \( M_1 \) is the \((1, 1)\) element of \( M \).

### 5. Motion analysis of n-link robot

We first characterize the invariant set \( W \) in (47) by analyzing the convergence value, \( V^* \), of the Lyapunov function \( V \) in (38). Then, we analyze the closed-loop equilibrium point.

#### 5.1. Convergence value of Lyapunov function \( V \)

Since \( \lim_{t \to \infty} V = 0 \) is equivalent to (11), we separately analyze two cases: \( V^* = 0 \) and \( V^* \neq 0 \).

**Case 1:** \( V^* = 0 \)

From (38) and (20), we have \( E^* = E_r, \tilde{q}_n = 0 \), and \( q_1^* = 0 \). Thus, from (47), we obtain

\[
q_n^* = \frac{2E_r}{M_1(q_1)} (1 - \cos q_1).
\]

Hence, as \( t \to \infty \), the closed-loop solution, \( (q(t), \dot{q}(t)) \), approaches the invariant set

\[
W_r = \left\{ (q, \dot{q}) \mid (q_1, \dot{q}_1) \text{satisfies (48)} : q_n = 0 \right\}.
\]

Since (48) is a homoclinic orbit (Sastry, 1999, p.44) with the equilibrium point \((q_1, \dot{q}_1) = (0, 0)\), \((q_1(t), \dot{q}_1(t)) \) has \((0, 0)\) as an \( \omega \)-limit point; that is, there exists a sequence of times \( t_m \) \( (m = 1, \ldots, \infty) \) such that \( t_m \to \infty = m \to \infty \) for which \( \lim_{m \to \infty} (q_1(t_m), \dot{q}_1(t_m)) = (0, 0) \). Therefore, there exists a sequence of times such that the robot enters any given small neighborhood of the upright equilibrium point.

**Case 2:** \( V^* \neq 0 \)

Putting \( E = E^*, q_n = q_n^*, \) and \( \tilde{q}_n = \tilde{q}_n^* \) into (39), we can show that \( E^* \neq E_r \), and that \( \tau \) is the constant vector \( \tau^* \) that satisfies

\[
k_B \tau^T \tilde{q}_n^* + (E^* - E_r) \tau^* = 0, \quad \text{with} \ E^* \neq E_r.
\]

In \( W \) described in (47), since \( q_n(t) = q_n^* \), we can consider links 2 to \( n \) to be a composite link; that is, the robot can be treated as an acrobat. The following two-step proof, which is similar to the one in Xin and Kaneda (2007a) for an acrobat, shows that \( q_1(t) \) is constant in \( W \):

**Step i.** Suppose the opposite, namely, that \( q_1(t) \) is not constant in the invariant set \( W \). Then, \( q_n(t) = q_n^* \), \( \tau(t) = \tau^* \), and (1) mean that \( \cos q_2^* = -1 \) and \( \tau_2^* = 0 \).

**Step ii.** Use (50) to obtain \( k_B \tilde{q}_2^* + (E^* - E_r) \tau_2^* = 0 \). This in combination with \( \tau_2^* = 0 \) shows that \( \tilde{q}_2^* = 0 \), which contradicts the statement \( \cos q_2^* = -1 \).

We now present a useful lemma.

**Lemma 1.** Consider the invariant set \( W \) defined in (47). Let \((q(t), \dot{q}(t)) \in W \). If \( V^* \neq 0 \), then \( q_1(t) \) is constant in the invariant set \( W \); that is, \( q(t) \equiv q^* \).

From \( q \equiv q^*, \tau \equiv \tau^* \), (1), and (8), we obtain

\[
G_1(q^*) = 0, \quad \tau^* = B^T G(q^*), \quad E^* = P(q^*).
\]

Define \( \Omega \) to be the closed-loop equilibrium set

\[
\Omega = \{(q^*, 0) \mid q^* \text{satisfies (50) and (51)}\}.
\]

We are ready for the following theorem.

**Theorem 2.** Consider the closed-loop system consisting of (1) and (44). Suppose that \( k_B \) satisfies (45), \( k_B > 0 \), and \( k_V > 0 \). Then, under controller (44), as \( t \to \infty \), the closed-loop solution \((q(t), \dot{q}(t))\) approaches

\[
W = W_r \cup \Omega, \quad \text{with} \ W_r \cap \Omega = \emptyset,
\]

where \( W_r \) is defined in (49), and \( \Omega \) is the set of equilibrium points defined in (52).

#### 5.2. Closed-loop equilibrium points

Note that all the closed-loop equilibrium points are in the set consisting of \( \Omega \) plus the upright equilibrium point. If \( \Omega \) contains an equilibrium point that is stable in the sense of Lyapunov stability, then the robot cannot be swung up arbitrarily close to the upright equilibrium point from some neighborhoods of that point.

Now, we analyze the relationship between \( \Omega \) and \( k_P \). Consider an equilibrium point \((q^*, 0) \) of \( \Omega \) in (52). From (50) and (51), we have

\[
G_1(q^*) = 0,
\]

\[
k_P \left( \tilde{q}_2^* + \sum_{j=2}^{n} \psi_j(q_2^*) \tilde{q}_j^* \right) + (P(q^*) - E_r) \tau_2^* = 0,
\]

\[
P(q^*) \neq E_r,
\]

where \( 2 \leq i \leq n \) holds in (55).

It is simple to check that \((q_1^*, \tilde{q}_2^*, \ldots, \tilde{q}_n^*) = (-\pi, 0, \ldots, 0) \) (or equivalently \((q_1^*, q_2^*, \ldots, q_n^*) = (-\pi, 0, \ldots, 0)\)), at which
\[ \tau_i^* = \cdots = \tau_n^* = 0 \text{, is a solution to (54)-(56) for any given } k_p. \]

In other words, at least one element of the set \( \Omega \) is the downward equilibrium point, where all the links are in the downward position. Based on this observation, our goal is to provide conditions on \( k_p \) that ensure that the set \( \Omega \) does not contain any other equilibrium point.

The following theorem is one of the main results of this paper. The proof is in the Appendix.

**Theorem 3.** Consider the closed-loop system consisting of (1) and (44). Suppose that \( k_D \) satisfies (45) and that \( k_V > 0 \). If \( k_p \) satisfies

\[ k_p > \max_{2 \leq i \leq n} k_{\text{mi}}, \quad (57) \]

where

\[ k_{\text{mi}} := 2E_i \beta_{i-1} \left( \sum_{j=i}^{n} \beta_j \right) / \left( \sum_{j=i-1}^{n} \beta_j \right), \quad (58) \]

then

1) \( \Omega \) in (52) contains only the downward equilibrium point \((-\pi, 0, \ldots, 0, 0, \ldots, 0)\);

2) the downward equilibrium point is unstable; and

3) the closed-loop solution \((q(t), \dot{q}(t))\) approaches

\[ W = W_i \cup \{(-\pi, 0, \ldots, 0, 0, \ldots, 0)\} \]

as \( t \to \infty \), where \( W_i \) is defined in (49).

5.3. Discussion

To obtain the relationship between \( k_p \) and the closed-loop equilibrium configuration \( q^* \), which satisfies (54)-(56), we use VCLs to iteratively study \( n-1 \) acrobat-type robots and thereby find conditions on \( k_p \) such that the set \( \Omega \) contains only the downward equilibrium point. Indeed, for the \( i \)-th \((2 \leq i \leq n)\) acrobat in Fig. 4, the second link is VCL \( i \), and the first link contains the links 1 to \( i-1 \) (which are shown stretched out in a straight line under the condition \( k_p > \max_{2 \leq j \leq i-1} k_{\text{mi}} \)) of the \( n \)-link robot.

To study (54)-(56), we consider the robot consisting of links 1 to \( i-1 \), and VCL \( i \). When \( q = q^* \), we can concisely rewrite \( P(q) \) in (5) as

\[ P(q^*) = \beta_1 \cos q_1^* + \cdots + \beta_{i-1} \cos(q_{i-1}^* + \cdots + q_{i-1}^*) + \beta_i^* \cos(q_{i-1}^* + \cdots + q_{i-1}^* + q_i^*), \quad (60) \]

where \( \beta_{i-1}^* \geq 2 \) is the value of \( \beta_i \) when \( q = q^* \). This enables us to rewrite \( G(q) \) in (7) with \( q = q^* \) as

\[ G_1(q^*) = -\beta_1 \sin q_1^* + \beta_i^* \sin(q_{i-1}^* + q_i^*), \quad (61) \]

\[ G_i(q^*) = \tau_i^* = -\beta_i^* \sin(q_{i-1}^* + \cdots + q_{i-1}^* + q_i^*), \quad (62) \]

It is crucial that, with the aid of these equations, we obtain an equation of the following form from (54) and (55):

\[ k_p \bar{q}_i^* = \eta_i(\bar{q}_i^*, \beta_i^*) \sin \bar{q}_i^*, \quad 2 \leq i \leq n, \quad (63) \]

where \( \eta_i \) is given in (A.19). Clearly, \( \bar{q}_i^* = 0 \) is a solution to (63). When \( \bar{q}_i^* \neq 0 \), we can rewrite (63) as

\[ k_p = \frac{\eta_i(\bar{q}_i^*, \beta_i^*) \sin \bar{q}_i^*}{\bar{q}_i^*}, \quad \forall \bar{q}_i^* \neq 0. \quad (64) \]

Since we can use Lemma A.1 in the Appendix to show that

\[ k_{\text{mi}} > \frac{|\eta_i(\bar{q}_i^*, \beta_i^*) \sin \bar{q}_i^*|}{|\bar{q}_i^*|}, \quad \forall \bar{q}_i^* \neq 0. \quad (65) \]

we can conclude that, if \( k_p > k_{\text{mi}} \), then \( \bar{q}_i^* = 0 \) is the only solution to (63).

However, if we do not use VCLs, then (15) and (16) will only yield an equation of the form

\[ k_p q_i^* = \xi_i(q_i^*), \quad 2 \leq i \leq n. \quad (66) \]

Since \( \xi_i \) does not have the property of (65), we still do not know how to find conditions on \( k_p \) that ensure that the only solution to (15) and (16) under the constraint \( P(q^*) \neq E_r \) is the downward equilibrium configuration.

Finally, we present a remark on Theorem 3.

**Remark 1.** The robot cannot actually remain at the downward equilibrium point since it is unstable in the closed-loop system. Thus, for almost all initial conditions, the closed-loop solution \((q(t), \dot{q}(t))\) approaches \( W_i \) as \( t \to \infty \). This shows that the objective of swing-up control can be achieved by the controller described above provided that the control parameters satisfy the conditions in Theorem 3.

6. Simulation results

Yeadon and Hiley (2000) developed a planar simulation model of a gymnast on the high bar that consists of four segments (arm, torso, thigh, leg) and a damped linear spring connecting the arm and torso. We used this 4-link model without the linear spring to verify our theoretical results. Table 1 shows the parameters.

<table>
<thead>
<tr>
<th>( m_i ) (kg)</th>
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<th>( k_i ) (m)</th>
<th>( f_i ) (kg m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.67</td>
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We can use Lemma A.1 in the Appendix to show that

\[ k_{\text{mi}} > \frac{|\eta_i(\bar{q}_i^*, \beta_i^*) \sin \bar{q}_i^*|}{|\bar{q}_i^*|}, \quad \forall \bar{q}_i^* \neq 0. \quad (65) \]

we can conclude that, if \( k_p > k_{\text{mi}} \), then \( \bar{q}_i^* = 0 \) is the only solution to (63).

However, if we do not use VCLs, then (15) and (16) will only yield an equation of the form

\[ k_p q_i^* = \xi_i(q_i^*), \quad 2 \leq i \leq n. \quad (66) \]

Since \( \xi_i \) does not have the property of (65), we still do not know how to find conditions on \( k_p \) that ensure that the only solution to (15) and (16) under the constraint \( P(q^*) \neq E_r \) is the downward equilibrium configuration.

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</table>
Figs. 5–7 show the simulation results for controller (44) with $k_D = 19450$, $k_P = 196500$, and $k_V = 6640$ for the initial condition $q(0) = [−7\pi/6, 0, 0, 0]^T$ and $\dot{q}(0) = [0, 0, 0, 0]^T$. Fig. 5 shows that $V$ and $E - E_r$ converge to 0; and Fig. 6 shows that $q_2$, $q_3$, and $q_4$ all converge to 0, and that several swings bring $q_1$ close to 0 or $-2\pi$. Notice that there exists a sequence of times during which the robot is swung up close to the upright equilibrium point. Finally, the time responses of the input torques $\tau_2$, $\tau_3$, and $\tau_4$ are presented in Fig. 7.

7. Conclusions

This paper concerns an $n$-link planar robot in a vertical plane with a passive first joint. We designed a swing-up controller that brings the robot into any arbitrarily small neighborhood of the upright equilibrium point, and we analyzed the global motion of the robot under the controller. Specifically, we first addressed the problem of how to devise a series of virtual composite links in an iterative way that could be used to design a coordinate transformation on the angles of all the active joints. Second, we constructed a Lyapunov function based on the transformation to design a swing-up controller. Third, we analyzed the global motion of the robot under the controller and established conditions on the control parameters that ensure attainment of the swing-up control objective. In fact, we provided the necessary and sufficient condition for the nonexistence of any singularities in the controller for any initial state. Moreover, to prevent the robot from becoming stuck at an undesired closed-loop equilibrium point, we used virtual composite links to clarify the relationship between the closed-loop equilibrium points and the control parameter $k_P$.

The results obtained in this study not only unify some previous results for an acrobot and a 3-link robot with a passive first joint, but also provide insight into the energy- and passivity-based control of underactuated multi-DOF systems. Moreover, this paper points the way toward a solution to the problem of controlling the motion when any one joint of the serial chain is not actuated. The notion of virtual composite links, as illustrated in this study, is expected to be applicable to control problems involving multi-link robots.

The stabilization, tracking, and robust control of $n$-DOF underactuated mechanical systems are interesting and challenging subjects for future study.

Acknowledgements

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Appendix. Proof of Theorem 3

To prove Theorem 1, we need the following lemma.

Lemma A.1. For the function $h(a, b, z)$ defined in (31) with $a > 0$ and $b \geq 0$, if $c \geq 0$ and $d \geq b$, then

$$b(h(a, b, z) + a + 2c + d) \sin z \leq \frac{2d(a + c + d)h(a, b, z)|z|}{a + d}. \quad (A.1)$$
Regarding statement 1), we carry out the proof by induction in two steps:

**Step 1:** For \( i = 2 \), we show that if

\[
k_0 > k_{m_2} = 2\beta_1 \sum_{j=2}^{n} \beta_j,
\]

then from (54) and (55), we obtain

\[
\begin{align*}
q_i^* &= 0, \quad \text{or} \quad q_i^* = -\pi, \\
\bar{q}_j &= 0, \quad \tau_j^* = 0, \quad \text{for } 2 \leq j \leq k
\end{align*}
\]

(4.3)

satisfies (54) and (55) with \( i = k \). For \( i = k + 1 \), we show that if \( k_0 > k_{m(k+1)} \), also holds, then the only solution to (55) for \( i = k + 1 \) is \( \bar{q}_{k+1} = 0 \) and \( \tau_{k+1}^* = 0 \).

This proves that, when (57) holds, only \((q_1^*, \bar{q}_2^*, \ldots, \bar{q}_n^*) = (0, 0, \ldots, 0) \) and \((q_1^*, \bar{q}_2^*, \ldots, \bar{q}_n^*) = (\mp \pi, 0, \ldots, 0) \) satisfy (54) and (55). Since the former contradicts (56), the latter is the only solution to (54)-(56).

Regarding **Step 1**, first, we use (61) to rewrite (54) as

\[
\beta_1 \sin q_1^* + \beta_2^* \sin(q_1^* + \bar{q}_2^*) = 0.
\]

(5.5)

\[
\tau_2^* \text{ in } (62) \text{ and } P(q^*) \text{ in } (60) \text{ yield}
\]

\[
k_0 \bar{q}_2^* - \beta_2^* P(q^*) - E_r \sin(q_1^* + \bar{q}_2^*) = 0,
\]

(6.5)

\[
\sin P(q^*) = \beta_1 \cos q_1^* + \beta_2^* \cos(q_1^* + \bar{q}_2^*).
\]

(7.5)

Adding \( P^2(q^*) \) to the square of (54) gives

\[
P^2(q^*) = h^2(\beta_1, \beta_2^*, \bar{q}_2^*).
\]

(8.5)

From (A.5) and (A.7), we obtain

\[
P(q^*) \sin(q_1^* + \bar{q}_2^*) = \beta_1 \sin \bar{q}_2^*.
\]

(9.5)

Therefore, if \( P(q^*) \neq 0 \), then we use (A.8) and (A.9) to eliminate \( q_1^* \) from (55) to obtain

\[
k_0 \bar{q}_2^* = \frac{ \beta_2^* (h(\beta_1, \beta_2^*, \bar{q}_2^*) - \text{sgn}(P(q^*))E_r) \sin \bar{q}_2^*}{h(\beta_1, \beta_2^*, \bar{q}_2^*)},
\]

(10.5)

where \( \text{sgn}(P(q^*)) \) denotes the sign of \( P(q^*) \). Now we show that \( P(q^*) \neq 0 \) under (A.2). First, suppose the opposite, namely, that \( P(q^*) = 0 \). Then, from (A.8), we have

\[
cos \bar{q}_2^* = -1 \quad \text{and} \quad \beta_1 = \beta_2^*.
\]

(11.5)

This reduces \( (A.6) \) to

\[
k_0 \bar{q}_2^* - \beta_1 E_r \sin \bar{q}_2^* = 0.
\]

(12.5)

From \( \cos \bar{q}_2^* = -1 \), we obtain \( \bar{q}_2^* = \pm \pi, \pm 3\pi, \ldots \). From \( \beta_1 = \beta_2^* \), it follows that the mechanical parameters of the robot must satisfy \( \beta_1 = \beta_2^* \leq \sum_{j=n}^{n-j} \beta_j \) and \( E_r = \beta_1 + \sum_{j=n}^{n-j} \beta_j \leq 2 \sum_{j=n}^{n-j} \beta_j \). Thus, under (A.2), we obtain

\[
k_0 \bar{q}_2^* \geq \beta_1 E_r \sin \bar{q}_2^* = 0.
\]

(13.5)

which contradicts (11.5). Thus, \( P(q^*) \neq 0 \) under (A.2).

Next, clearly \( \bar{q}_2^* = 0 \) is a solution to (10.5) for any \( \beta_2^* \). To apply Lemma A.1, we define \( z = \bar{q}_2^*, a = \beta_1, b = \beta_2^*, c = 0 \), and

\[
d = \sum_{j=2}^{n} \beta_j.
\]

That gives us \( E_r = a + d \) and \( d \geq b \). Thus, when \( \bar{q}_2^* \neq 0, \) we can rewrite (A.10) as

\[
k_0 \bar{q}_2^* = \frac{b (h(a, b, z) - \text{sgn}(P(q^*)) (a + d)) \sin z}{h(a, b, z)z}.
\]

(14.5)

It follows directly from Lemma A.1 that

\[
\frac{b(h(a, b, z) + a + d)}{h(a, b, z)z} \leq \frac{2(a + d)}{a + d} = \frac{k_{m_2}}{\beta_1}.
\]

(15.5)

Thus, only \( \bar{q}_2^* = 0 \) satisfies (10.5) under (A.2).

When \( \bar{q}_2^* = 0, (A.5) \) yields \( \sin \bar{q}_2^* = 0 \). This gives us \( \tau_2^* = \frac{q_2^*}{G^2(q^*)} = -\beta_2^* \sin(q_1^* + \bar{q}_2^*) = 0 \), thereby showing that (4.3) is true. Moreover, from (A.7), we have

\[
\begin{align*}
P(q^*) &= \beta_1 \beta_2^* > 0, \quad \text{if} \quad q_1^* = 0, \\
P(q^*) &= -\beta_1 \beta_2^* < 0, \quad \text{if} \quad q_1^* = -\pi.
\end{align*}
\]

(16.5)

Regarding **Step 2**, since \( q_2^* = \cdots = \bar{q}_k^* = 0 \) owing to (A.4), we can simplify (55) for \( i = k + 1 \) to

\[
k_0 \bar{q}_k^* + (P(q^*) - E_r) \bar{q}_k^* = 0.
\]

(17.5)

Using (A.4), (A.14), and (P(q^*) in (360), we obtain

\[
P(q^*) = \beta_1 \beta_2^* \cdots \beta_k^* \bar{q}_k^*.
\]

(18.5)

We can use the formula for \( \sin \theta_{k+1} \) in (26) and \( \tilde{\beta}_k^* \) in (15) to reduce (A.15) to

\[
k_0 \bar{q}_k^* = \frac{\tilde{\beta}_k^* (|P(q^*)| - \text{sgn}(P(q^*))E_r) \sin \bar{q}_k^*}{h(\beta_k, \tilde{\beta}_k^*, \bar{q}_k^*)}.
\]

(19.5)

Clearly, \( \bar{q}_k^* = 0 \) is a solution to (19.5). To use Lemma A.1, we define \( z = \bar{q}_k^*, a = \beta_k, b = \beta_k^*, c = \sum_{j=k}^{k-1} \beta_j \), and

\[d = \sum_{j=k+1}^{n} \beta_j.
\]

That gives us \( d \geq b \) and \( E_r = a + c + d \). When \( \bar{q}_k^* \neq 0 \), we rewrite (A.19) as

\[
k_0 \bar{q}_k^* = \frac{b (h(a, b, z) - \text{sgn}(P(q^*)) (a + 2c + d)) \sin z}{h(a, b, z)z}.
\]

(20.5)

It follows directly from Lemma A.1 that

\[
\frac{b(h(a, b, z) + a + 2c + d)}{h(a, b, z)z} \leq \frac{2de_r}{a + d}.
\]

(21.5)

Thus, if \( k_0 > k_{m(k+1)} \), then (A.20) has no solution; that is, (19.5) has the unique solution \( \bar{q}_k^* = 0 \). This yields \( \theta_{k+1}^* = 0 \), thus completing **Step 2**.

Regarding statement 2), letting \((-\pi + \delta, 0, 0, \ldots, 0)\) be a point in a neighborhood of the downward equilibrium point yields \( V(-\pi + \delta, 0, 0, \ldots, 0) \) is an increasing function under controller (44), according to Theorem 2, no matter how small \( |\delta| > 0 \) is, an \( n \)-link robot starting from \((-\pi + \delta, 0, 0, \ldots, 0)\) will not approach \((-\pi, 0, 0, \ldots, 0)\), but will approach \( W \). Instead, this shows that the downward equilibrium point is unstable.

Regarding statement 3), it is a direct consequence of statement 1) and Theorem 2. ■
References


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